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Neutrino Magnetic Moment*

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We review attemps to achieve a large neutrino magnetic moment ($\mu_\nu \geq 10^{-11}\mu_B$), while keeping neutrino light or massless. The application to the solar neutrino puzzle is discussed.

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INTRODUCTION

The so called solar neutrino puzzle is now more than twenty years old. The experimenrally detected[1] flux of neutrinos originating from the thermonuclear reactions in the sun via the process

$$\nu_e \, ^{37}C\ell \rightarrow e^- \, ^{37}A_r \tag{1}$$

is far below the theoretical predictions.[2] Whereas the standard Solar Model (SSM) suggests the conversion rate

$$< {}^{37}Cl >_{SSM} = (7.9 \pm 2.6) \text{ SNU} \tag{2}$$

the experiments give a rate roughly a factor of three smaller

$$< {}^{37} C\ell >_{exp} = (2.1 \pm 0.3) \quad \text{SNU} \tag{3}$$

The unit 1 SNU (Solar Neutrino Unit) corresponds to 10^{-36} captures/atom-sec. For the experiment of Davis et al 1 SNU is equivalent to the conversion of about 0.23 atoms of $^{37}A_r$ per day, so that (3) gives a signal of about 0.5 atoms per day. This is a rather challenging experiment and we won't go into the uncertainties associated with it or the theory, we suggest Peccei's review for a nice discussion of whether this is a true paradox or not. The beauty of this issue is it touches upon the fundamental properties of neutrinos such as their masses, mixing angles,magnetic moments etc.

The most popular explanation to date is the MSW mechanism[4] of resonant oscillations in the interior of the sun, which transform ν_e into ν_μ (or ν_τ). For this to work, the masses and the mixing angles are constrained by

$$\begin{aligned} \Delta m^2 = m^2_{\nu_\mu} \; (m^2_{\nu_\tau}) - m^2_{\nu_e} \sim 10^{-7} - 10^{-4} \, (eV)^2 \\ \sin^2 \theta \;\geq\; 10^{-3} \end{aligned} \tag{4}$$

We should stress that the MSW process takes place in the radiation zone ($0.04 - 0.7 \, R_\odot$, where $R_\odot$ is the radius of the sun). This perfectly acceptable explanation needs further laboratory verification, for we still have no clue on the values of neutrino masses and mixing angles. A potential death blow to this idea could come, however, from the very observations of Davis et al. They seem to suggest an anticorrelation between solar neutrino flux and the sun-spot activity. At the sunspot maximum, the neutrino flux is below average and at sun-spot minimum activity it is far above the average value: (5.1 ±1) SNU (1987-1988). The sun-spot activity is attributed to magnetic storms in the convection zone of the sun, near the surface (0.7 - 1 $R_\odot$), when the magnetic field near the maximum reaches the value of 10^3 - 10^4 Gauss. If this is so, then the mass oscillation cannot be the explanation; it has to do with the magnetic field of the sun.

Cisneros[5] and Voloshin[6] et al (VVO) suggests that the electron neutrino possesses a large magnetic moment which enables it to flip into the sterile right - handed component (or other flavors) in the magnetic field of the sun. This seems to work if the magnetic moment is on the order of $10^{-11}\mu_B$ ($\mu_B = \frac{e}{2m_e}$) , which, as we will see below, is an enormous number, many orders of magnitude above the value in the standard model (with ν_R).

NEUTRINO MAGNETIC MOMENT

Recall that, like the mass, magnetic moment is the helicity flip operator

$$\mu_\nu \bar{\nu}_R \sigma^{\mu\nu} \nu_L F_{\mu\nu} = \mu_\nu \nu_L^T C \sigma^{\mu\nu} \nu_L^C F_{\mu\nu} \tag{5}$$

For the VVO mechanism to work, μ_ν is estimated[6]

$$\mu_\nu \sim 10^{-11} - 10^{-10} \mu_B \tag{6}$$

To get a feeling for this number, let us campare it with the natural value for μ_ν in the standard model[7] (with ν_R)

$$(\mu_\nu)_{st} = \frac{3eG_F m_\nu}{8\sqrt{2}\ \pi^2} = 3.2 \times 10^{-19} \left(\frac{m_\nu}{1ev} \right) \mu_B \qquad (7)$$

which is at least eight orders of magnitude smaller than the VVO number! This number can get substantially enhanced if there is a right-handed current, such as in left-right models[8)]

$$(\mu_\nu)_{LR} = \frac{G_F\ m_e}{2\sqrt{2}\ \pi^2} \sin 2\Phi \simeq 2 \times 10^{-13} \sin 2\Phi\ \mu_B \qquad (8)$$

where Φ is the mixing angle between left and right-handed currents, $|\Phi| \leq 0.05$. Obviously, (8) is still too small. With some optimal assumptions supersymmetry may give even larger prediction, but still

$$(\mu_\nu)_{SS} \lesssim 10^{-12}\ \mu_B \qquad (9)$$

Thus we have problem at our hands: how to reconcile a large value for μ_ν with a small neutrino mass, $m_\nu \lesssim 10eV$? This is quite a challenge for a model builder, one which is hard to ignore, even if you are not very convinced with the VVO mechanism. The idea, of course, is to uncorrelate μ_ν with m_ν, which was achieved by Babu and Mathur and Fukugita and Yanagida.[9)] They introduce an SU(2) singlet, charged Higgs field h^+, which works OK for μ_ν. The trouble is that in their models neutrino has a bare Dirac mass, which has to be fine tuned to be small.[10)] This is not a solution to our problem.

An ingenious way out was suggested by Voloshin[11)], who postulated an $SU(2)_H$ symmetry between ν and ν^c. Under $SU(2)_H$ the mass term behaves as a triplet and so is forbidden, whereas the magnetic moment interaction is allowed. Namely,

$$\begin{aligned} &L_\nu^T\ i\ \sigma_2\ CL_\nu = 0 \\ &L_\nu^T\ i\ \sigma_2\ C\ \sigma_{\mu\nu} L_\nu \neq 0 \end{aligned} \qquad (10)$$

where $L_\nu = \binom{\nu}{\nu^C}_L$. This idea, unfortunately is not easy to realize, since $SU(2)_H$ does not commute with $SU(2)_L$. You could try to enlarge $SU(2)_L \times U(1)$ a la Barbieri and Mohapatra[12], who use $SU(3)_L \times U(1)$ with the basic triplet consisting of e, ν and ν^C. This automatically contains $SU(2)_H$, but the scale of $SU(2)_H$ symmetry breaking is pushed above M_W! This means that $SU(2)_H$ effectively loses its original role; we are back where we started.

This may be a good place to mention some potential troubles with the large value of μ_ν.

(a) laboratory experimental[13] limit on μ_ν is

$$(\mu_\nu)_{exp} \leq 1.5 \times 10^{-10} \mu_B \tag{11}$$

(b) from the stellar cooling analysis, there is an astrophysics bound[14]

$$(\mu_\nu)_{astro} \leq 1.1 \times 10^{-11} \mu_B \tag{12}$$

(c) also, cosmological argument due to increase in ν_L number from the $\nu_R + e \rightarrow \nu_L + e$ process, induced via μ_ν, implies[14]

$$(\mu_\nu)_{cosmol} \leq 0.5 \times 10^{-11} \mu_B \tag{13}$$

(d) and finally, the energy loss of the supernovae 1987a would be too large, unless[15]

$$(\mu_\nu)_{sn} \lesssim 10^{-12} \mu_B \tag{14}$$

Obviously the serious problem is d) only, but there one could try to add new interactions which would trap ν_R, so even this is not a fatal blow to the VVO suggestion. Still, we find the idea of transition magnetic moments far more appealing. Here, in analogy with neutrino oscillations, one takes $\nu_{\mu L}$ instead of ν^C_{eL}. In other words, one assumes the horizontal symmetry[16] $SU(2)_H$ between the Weyl fields ν_e and ν_μ, so that now

$$L_\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L \tag{15}$$

The invariant magnetic moment (10), with L_ν from (15) is now a transition moment

$$\nu_e^T C\sigma_{\mu\nu} \nu_\mu F^{\mu\nu} \neq 0 \qquad (16)$$

The diagonal moments vanish in the case of Weyl (Majorana) neutrinos.

The new problem appears, though; the magnetic field of the sun has no energy to flip the neutrinos, unless the mass difference between ν_e and ν_μ is almost vanishing[17)]

$$\Delta m^2 = m^2_{\nu_\mu} - m^2_{\nu_e} \lesssim 10^{-7} (\mathrm{eV})^2 \qquad (17)$$

The model building becomes even more challenging. The most natural approach is to use some symmetry, such as U(1) global symmetry $L_e - L_\mu$, which would imply $\Delta m^2 = 0$. In this case, ν_e and ν_μ group together into a Dirac neutrino. However, we still have to achieve $m_\nu \lesssim 10$eV or so.

Now $SU(2)_H$ could be either global or local, however, in both cases its scale of symmetry breaking M_H must be large. If it is global, the consequence of Goldstone bosons implies $M_H \geq 10^7 - 10^8$ GeV, and the local symmetry case demands $M_H > M_W$. But this defies the whole purpose for imposing $SU(2)_H$ in the first place; we need this symmetry at low energies, so that it can play its custodial role. Ideally, $M_H << 1$ GeV is what we are after. We are back where we started. What to do?

A suggestion by Leurer and Marcus[17)] is to have an approximate $SU(2)_H$, broken explicitly. Encouraged by the fact that in the standard model $SU(2)_H$ is broken only by $\varepsilon = \frac{m_\mu - m_e}{M_H} \sim 10^{-4}$ terms, they demand the breaking of this symmetry to continue being proportional to ε, a rather ad hoc assumption. This would enable them to lower the value of

m_ν down to eV scale.

Leurer and Golden[19] try a U(1), instead of $SU(2)_H$ symmetry. This also has its problems, in particular they need some further fine tuning to achieve a small mass for m_ν.

Yet another possibility is to use a discrete symmetry. The discrete symmetry certainly needs a serious motivation, for it carries a potential disaster of domain walls. To get rid of this, we appeal to the idea of possible symmetry non-restoration at high temperature, as advocated by Weinberg and especially Mohapatra and G.S.[20] (or soft breaking, if you find the above hard to digest). The main advantage of the discrete symmetry is that it can be kept down to as low energies as we wish (maybe even left unbroken?) Besides us, the discrete symmetry approach was also tried by Babu and Mohapatra[21] and by the Wienna group[22]. Babu and Mohapatra argue that the combination of supersymmetry and the Z_4 subgroup of $L_e - L_\mu$ works, whereas in the case of Wienna group Dirac neutrino ends up light for somewhat accidental resons (but technically natural). The space doesn´t permit as to discuss their work at great length; for reasons that you will probably find obvious we shall devote the rest of this paper to our work.

THE DISCRETE SYMMETRY AND μ_ν

The first fact to notice is that the custodial symmetry must be nonabelian, if it is to forbid $\nu_i^T C \nu_j$ and allow $\nu_i^T C \sigma_{\mu\nu} \nu_j$. The simplest possibility would be a subgroup of Voloshin´s $SU(2)_H$ symmetry which does the job. The ideal candidate for this is the quaternion symmetry Q_4 of 8 elements, whose characteristics are best read off from its faithfull two dimensional representation

$$\{ \pm 1_2, \ \pm i \ \sigma_i \}$$

(a) Quaternion symmetry Q_4

Since there is always a trivial repr. (R_1), obviously there are four one dimensional representations $R_1, \dots R_4$. The five classes ($C_1 = \{1_1\}$, $C_2 = \{-1_2\}$, $C_3 = \{\pm i\sigma_1\}$, $C_4 = \{\pm i\sigma_2\}$, $C_5 = \{\pm i\sigma_3\}$ in the case of a doublet representation D) are then given by the character table

	C_1	C_2	C_3	C_4	C_5
R_1	1	1	1	1	1
R_2	1	1	1	-1	-1
R_3	1	1	-1	1	-1
R_4	1	1	-1	-1	1
$R_5 = D$	2	-2	0	0	0

Following Voloshin, we assume $L_\nu = \binom{\nu_e}{\nu_\mu}$ to be in a doublet representation of Q_4. In the same manner as with $SU(2)_H$ symmetry (see eq. (10)), the mass term is forbidden, whereas magnetic moment is allowed. The point is that the antisymmetric transformation $D^T i\sigma_2 D$ is invariant, which vanishes for m_ν. One can now construct one´s favourite model; all you need the decomposition

$$(D \times D)_{AS} = R_1$$

$$(D \times D)_S = R_2 + R_3 + R_4$$

Obviously, both new fermions and Higgs fields are needed, and it is somewhat a matter of taste of what to choose.

We shall not describe the details of our model here[23), for they are given in our paper. Let us rather concentrate on the shortcomings of the Q_4 approach. The main problem is the fine tuning needed to keep Δm_ν^2 small. The way we construct the model, there is no protective symmetry which could make ν_e, ν_μ a Dirac neutrino. The potential candidate is a Z_4 symmetry generated by, say $i\sigma_1$ (or any other $i\,\sigma_k$), which should remain unbroken. Therefore, only those Higgs fields transforming as R_1 or R_2 are allowed nonvanishing ver´s. This forces e_R, μ_R to form a doublet D and the outcome is that we cannot split e, μ masses. This problem seems to remain even if we add additional particles.

(b) <u>dicyçlic group Q_6</u>

This group consisting of 12 elements is more promising, since it has two doublet representations. It is generated by elements r,s through

$$r^6 = s^4 = 1, \quad srs = r^2 \tag{18}$$

Its six classes $C_1\{1\}$, $C_2\{-1\}$, $C_3\{r^2,-r\}$, $C_4\{s,-sr,sr^2\}$, $C_5\{-s,sr,-sr^2\}$, $C_6\{r,-r^2\}$ has the following character table

	C_1	C_2	C_3	C_4	C_5	C_6
R_1	1	1	1	1	1	1
R_2	1	1	1	-1	-1	1
R_3	1	-1	1	i	-i	-1
R_4	1	-1	1	-i	i	-1
R_5	2	-2	-1	0	0	1
R_6	2	2	1	0	0	-1

The explicit form of r and s for R_5,R_6 is

$$r_5 = \exp\left(\frac{i\ 2\pi\sigma_1}{6}\right), \quad s_5 = i\sigma_3$$

$$r_6 = \exp\left(\frac{i\ 2\pi\sigma_1}{3}\right), \quad s_6 = -\sigma_3 \tag{19}$$

It is clear that Q_6 is not a subgroup of $SU(2)_H$, but rather $U(2)_H$. However, it still forbids the neutrino mass, if we assign left-handed leptons into R_5 (or $(R_5)^+$).

Our strategy is straightforward: we break Q_6 down to Z_4 generated by s. This implies neutrino mass term $\nu_e^T C \nu_\mu$, or $\Delta m_\nu^2 = 0$. Furthermore, if you choose

$$e_R(R_4), \quad \mu_R(R_2)$$

$$\psi = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix} \quad (R_5)^+ \tag{20}$$

then under s

$$e_{L,R} \rightarrow -i\, e_{L,R}\ ; \quad \mu_{L,R} \rightarrow i\, \mu_{L,R} \tag{21}$$

In other words, s is a Z_4 subgroup of $L_\mu - L_e$. The dangerous processes such as $\mu \rightarrow e\gamma, \mu \rightarrow ee\bar{e}$ are forbidden.

Besides the usual particles, this approach needs additional lepton doublets $N_{L,R}$ and new Higgs fields in order to generate large μ_ν. As in our Q_4 work[23)], the neutrino (now Dirac) mass is suppressed by m_Q/M_W, where m_Q is the scale of Q_6 symmetry breaking. In order to split $m_\mu - m_e$, $m_Q \simeq m_\mu$, giving us m_ν in the eV region.

The technical details of our model will be spelled out elsewhere[24], together with the phenomenological implications. It's major fault is the proliferation of new fields, but it seems to work. We hope, though, that a more elegant solution along these lines will soon emerge. Needless to say, new experimental results regarding the sun-spot activity and neutrino solar flux are badly needed to tell us whether the anticorrelation between the two is real.

REFERENCES AND FOOTNOTES

1. R.Davis,T.Cleveland and J.K.Rowley, in Steamboat Springs 1984, Proceedings,Interactions Between Particle and Nuclear Physics, pp 1037-1050
2. I.N.Bahcall et al, Rev.Mod.Phys. 54,767(1982); I.N.Bahcall and R.K.Ulrich,Rev.Mod.Phys.60,297(1988)
3. R.D.Peccei, The physics of neutrinos, DESY preprint 89-043 (1989). This nice review of neutrino physics offers also a comprehensive set of references.
4. L.Wolfenstein, Phys.Rev. D17,2369(1978);D20,2634(1979); S.P.Mikheyev and A.Yu.Smirnov, Nuovo Cim. 9C,17(1986)
5. A.Cisneros, Astrophys.Space Sci. 10,87(1981)
6. L.B.Okun,M.B.Voloshin and M.I.Vysotsky,Soc.J.Nucl.Phys.91, 754(1986);Sov.Phys. JETP 64,446(1986)
7. K.Fujikawa and R.E.Shrock,Phys.Rev.Lett.45,963(1980); B.W. Lynn,Phys.Rev. D23,2151 (1981)
8. J.E.Kim, Phys.Rev.D14,3000(1976); M.A.B.Bég,W.J.Marciano and M.Ruderman,Phys.Rev.D17,1395(1978); R.E.Shrock,Nucl.Phys. B206,359(1982);
J.Liu,Phys.Rev. D35,3447 (1987)
M.J.Duncan et al, Phys.Lett. 191B,304(1987)
9. K.S.Babu and V.Mathur,Phys.Lett. 196B,218(1987);M.Fukugita and T.Yanagida,Phys.Rev.Lett. 58,1807(1987)
10. J.Pulido and J.Ralston,Phys.Rev. D38,2864(1988)
J.Liu,Phys.Lett. 225B, 148(1989)
11. M.Voloshin,Sov.J.Nucl.Phys. 48,512 (1988)
12. R.Barbieri and R.N.Mohapatra, Phys.Lett. 218B,225(1989)
13. A.V.Kyuldjiev, Nucl.Phys. B243, 387 (1984)
14. M.Fukugita and S.Yazaki,Phys.Rev. D36,3817(1987)
15. R.Barbieri and R.N.Mohapatra, Phys.Rev.Lett. 61,27(1988)
J.Lattimer and J.Cooperstein, ibid 61,23(1988)
16. K.S.Babu and R.N.Mohapatra,Phys.Rev.Lett.63,228(1989)
17. E.Akhmedov,Phys.Lett. 213B,64(1988);
C.-S.Lim and W.J.Marciano,Phys.Rev. D37,1368(1988)
18. M.Leurer and N.Marcus,Technion preprint PH-89-48(1989)
19. M.Leurer and M.Golden,Fermilab preprint 89/175-T(1989)
20. S.Weinberg,R.N.Mohapatra and G.Senjanović,Phys.Rev.D20,3390 (1979)
21. K.S.Babu and R.N.Mohapatra, Phys.Rev.Lett.
22. G.Ecker, W.Grimus, H.Neufeld,CERN preprint - TH.5485/89
23. D.Chang,W.-Y.Keung and G.Senjanović,Phys.Rev.D (to be publ.)
24. D.Chang,W.-Y.Keung,S.Lipovača and G.Senjanović,to appear